A comparative analysis of algorithms for the magnetoencephalography inverse problem

A Sorrentino[1], A Pascarella[2], C Campi[3] and M Piana[2]

[1] CNR-INFM LAMIA, Genova, Italy

[2] Dipartimento di Informatica, Università di Verona, Cà Vignal 2, Strada le Grazie 15, 37134, Verona, Italy

[3] Dipartimento di Matematica, Università di Genova, via Dodecaneso 35, 16146, Genova, Italy

E-mail: sorrentino@fisica.unige.it

Abstract. We present a comparison of three methods for the solution of the magnetoencephalography inverse problem. The methods are: an eigenspace projected beamformer, an algorithm implementing multiple signal classification with recursively applied projection and a particle filter for Bayesian tracking. Synthetic data with neurophysiological significance are analyzed by the three methods to recover position and amplitude time course of the active sources.

1. Introduction

Magnetoencephalography (MEG) records the magnetic fields due to the brain activity in a non-invasive way and with a high temporal resolution (about 1 ms) [1]. Due to this outstanding temporal resolution, MEG is particularly suited for investigating how different brain functions interact when complex tasks are performed. Although the simple analysis of the temporal evolution of the magnetic field distribution (measured by a helmet-shaped array of sensors) can give approximate information on the active brain regions, the full exploitation of the information content of the data requires the solution of an inverse problem which consists in recovering the spatio-temporal evolution of the neural currents from the dynamical measurements of the magnetic field at different locations outside the skull. However solving the MEG inverse problem is a complicated task for a couple of reasons: as most inverse problems, the MEG problem is ill-posed in the sense of Hadamard [2]; it is a dynamical problem due to the high sampling frequency of the data; finally, data are usually highly corrupted by noise, of both environmental and neural origin.

In the last twenty years, a considerable amount of papers have been published, which apply several methods to the solution of the MEG inverse problem; suggested methods range from the Levenberg-Marquardt algorithm [5] to the L2 and L1 regularization [6, 7], from genetic algorithms [4] to nonlinear optimization techniques borrowed from the signal processing community, such as RAP-MUSIC [8], to spatial filters (beamformers) [9]; even more recently a Bayesian tracking approach has been suggested in [13] and applied in [14] and [15].

We perform a comparative analysis of three approaches which seem to share most of the properties an "ideal" algorithm should have: automaticity, numerical effectiveness, robustness with respect to high levels of noise and generality with respect to the temporal evolution of active sources. In particular, RAP-MUSIC discriminates the signal from the noise by a singular value decomposition analysis of the correlation matrix of the data and accounts for time correlation by investigating spaces of increasing dimension; the beamforming algorithm spatially filters the signal to focus the source as a weighted combination of the measurements; particle filters realize a Bayesian tracking of the sources by means of a sequential Monte-Carlo sampling-resampling of the probability density functions involved. From the analysis of the assumptions and principles behind each method, we select a set of neurophysiologically plausible synthetic experimental conditions which could highlight the limits of the analyzed methods. For estimating the accuracy of the reconstructions we compute the localization error and we compare the reconstructed source strengths with the original ones.

2. The MEG inverse problem

Within the quasi-static approximation, a neural current $\mathbf{j}(\mathbf{r}',t)$ inside the brain volume Ω produces a magnetic field given by the Biot-Savart equation with the addition of an ohmic current term:

$$\mathbf{b}(\mathbf{r},t) = \frac{\mu_0}{4\pi}\int_\Omega [\mathbf{j}(\mathbf{r}',t) + \mathbf{j}^{ohm}(\mathbf{r}',t)] \times \frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^3} d\mathbf{r}' \tag{1}$$

where $\mathbf{b}(\mathbf{r},t)$ is the magnetic field at $\mathbf{r}$ at time t, $\mathbf{j}^{ohm}(\mathbf{r}',t)$ is the ohmic or volume current inside the brain (generated by the electric potential of the neural current itself) and μ_0 is the magnetic permittivity of vacuum. The forward problem is then twofold: first, it is necessary to find the ohmic current distribution elicited by a certain neural current distribution; then, the Biot-Savart equation has to be computed to obtain the measurable field. In the applications, the neural current distribution is either discretized on a dense set of points inside the brain volume, or modelled as a small set of point-wise currents; therefore in both cases solving the forward problem for a single point-wise current (or current dipole) of the form

$$\mathbf{j}(\mathbf{r}) = \mathbf{q} \cdot \delta(\mathbf{r} - \mathbf{r_q}) \quad , \tag{2}$$

where $\mathbf{q}$ is the dipole moment and $\mathbf{r_q}$ the dipole position, is enough. In general, the computation of the forward problem is influenced by the geometry and the conductivity of the conductor (of the head): for a piecewise homogeneous spherically symmetric conductor analytic results are available [10], while for more general models Boundary Element Methods or Finite Element Methods are used.

3. Algorithms

The number of methods proposed and applied for solving the MEG inverse problem is too high to allow for a comprehensive summary here. However, the number of methods which can automatically and reliably recover point sources without a large number of false positives, and is therefore suitable for practical use, is not that high. Among the practically usable algorithms, we briefly review two of the most used ones, an eigenspace projected (EP) beamformer [12] and RAP-MUSIC [8], and one more recently proposed, particle filters [13].

3.1. Eigenspace projected beamformer

Model and assumptions. The neural current is modelled as a continuous current distribution and discretized on a dense set of points $\{\mathbf{r}_n\}_{n=1,\dots,N}$, in the following also referred to as grid. The Biot-Savart equation becomes a linear matrix equation:

$$\mathbf{B} = \mathbf{G} \cdot \mathbf{J} \tag{3}$$

where the matrix $\mathbf{B}$ contains the spatio-temporal pattern of the measured magnetic field, $\mathbf{G}$ is the forward matrix, i.e. the ordered collection of forward sub-matrices $\mathbf{G}(\mathbf{r}_n)$ each one containing the forward solution for three orthogonal unit dipoles at a given point $\mathbf{r}_n$ of the grid, and $\mathbf{J}$ contains the temporal evolution of the components of the current distribution on the grid points $\mathbf{J}(\mathbf{r}_n)$. For applying the EP beamformer, the following facts are assumed: (i) the number of brain sources is small, so that a principal component analysis of the measurements is feasible, (ii) the spatial configuration of neural sources is stationary and (iii) sources are not time-correlated (their time courses are linearly independent).

Algorithm. Beamformers are spatial filters discriminating the signals on the basis of their spatial location. The output of an EP beamformer at point $\mathbf{r_n}$ of the computational grid discretizing the brain volume is

$$\hat{\mathbf{J}}(\mathbf{r}_n) = \mathbf{W}(\mathbf{r}_n)^\top \Pi \, \mathbf{B} \tag{4}$$

where $\mathbf{W}(\mathbf{r}_n)^\top$ is the beamformer matrix and Π is a projector onto the eigenspace (also referred to as signal subspace) generated by the biggest principal components of the data matrix. The weight matrices $\mathbf{W}(\mathbf{r}_n)$ are the key unknowns in beamforming and can be determined by solving the constrained minimum problem

$$\min_{\mathbf{W}(\mathbf{r}_n)} \mathrm{var}\hat{\mathbf{J}}(\mathbf{r}_n) \quad \text{subject to} \quad \mathbf{W}(\mathbf{r}_n)^\top \mathbf{G}(\mathbf{r}_n) = I_3 \tag{5}$$

which, with a regularized inversion for the data covariance matrix, leads to

$$\mathbf{W}(\mathbf{r}_n) = (\mathbf{B}\mathbf{B}^\top + \lambda \mathbf{I})^{-1} \mathbf{G}(\mathbf{r}_n) \left[\mathbf{G}^\top(\mathbf{r}_n) (\mathbf{B}\mathbf{B}^\top + \lambda \mathbf{I})^{-1} \mathbf{G}(\mathbf{r}_n) \right]^{-1} \tag{6}$$

Under the previously stated assumptions it can be shown that finding the weight matrix $\mathbf{W}(\mathbf{r}_n)$ which minimizes (5) corresponds to finding the source $\hat{\mathbf{J}}(\mathbf{r}_n)$ with the strength closest to the strength of $\mathbf{J}(\mathbf{r}_n)$.

Parameters. Using an EP beamformer, the free parameters the user needs to define are:

- the dimension of the signal subspace, i.e. the number of eigenvectors of $\mathbf{B}\mathbf{B}^\top$ to be used in the projector Π;
- the regularization parameter λ in the inversion of the data covariance matrix.

3.2. RAP-MUSIC

Model and assumptions. The neural current is modelled as a small set of current dipoles; dipoles positions and orientations are assumed to be fixed during the measurements sequence, and only the dipole strengths (the norm of $\mathbf{q}$) are allowed to vary; therefore, the following factorization for the magnetic field makes sense:

$$\mathbf{B} = \mathbf{A} \cdot \mathbf{S} \tag{7}$$

where $\mathbf{A}$ contains the forward solution for the small set of current dipoles, while $\mathbf{S}$ contains the dipole strengths as functions of time. We remark that, although equations (3) and (7) look rather similar, the matrix $\mathbf{G}$ is known once a discretization for the neural current is chosen, while matrix $\mathbf{A}$ is unknown as the positions and orientations of the dipolar sources are the key unknowns of the problem. Furthermore, while the matrix $\mathbf{G}$ contains, for each grid point, three forward vectors corresponding to the three orthogonal directions, in matrix $\mathbf{A}$ also the orientation is given, then only one forward vector is contained for each current dipole. The assumptions behind RAP-MUSIC (small number of spatially stationary sources) resemble those for the EP beamformer.

Algorithm. RAP-MUSIC is a recursive extension of the MUSIC algorithm which estimates the parameters of operator $\mathbf{A}$ from an estimate of its range, and then obtains through a standard least-squares approach the time behaviour of the sources coded in $\mathbf{S}$. The key idea is to exploit the covariance matrix of the measurements:

$$\mathbf{B}\mathbf{B}^{\top} = \mathbf{A}\mathbf{S}\mathbf{S}^{\top}\mathbf{A}^{\top} \quad . \tag{8}$$

It can be shown that (i) the number of eigenvalues of $\mathbf{B}\mathbf{B}^{\top}$ greater than zero (or greater than the noise variance σ^2, when a noisy framework is considered) is an estimate of the number of sources (where by "source" we mean a single dipole or a couple of correlated dipoles); (ii) the subspace generated by the eigenvectors associated with these eigenvalues (signal subspace) is an estimate of the range of $\mathbf{A}$. Then the MUSIC algorithm is realized in three steps: (i) $\mathbf{B}\mathbf{B}^{\top}$ is diagonalized, an estimate of the number of sources is made from the plot of the eigenvalues and the signal subspace is computed; (ii) all the points in the brain and, for each point, all the orientations, are spanned to find those which give the highest subspace correlation [3] with the signal subspace; if the subspace correlation exceeds a given threshold (often set to 0.95) the dipole is accepted and a search for a new dipole begins, otherwise a new search begins for a couple of correlated sources (by spanning all the couples of points and orientations).

Parameters. Using RAP-MUSIC, the free parameters the user needs to define are:

- the dimension of the signal subspace, i.e. the number of sources;
- the threshold of the subspace correlation, roughly corresponding to the desired goodness of fit.

3.3. Particle filters

Model and assumptions. The neural current is modelled as a small set of point sources; the number of sources, as well as their positions and moments, may vary with time:

$$\mathbf{j}(\mathbf{r}, t) = \sum_{i=1}^{M(t)} \mathbf{q}^i(t) \cdot \delta(\mathbf{r} - \mathbf{r}_q^i(t)) \tag{9}$$

so that the neural currents form a stochastic process, which is assumed to be a Markov process [11]. It is also assumed that the sequence of magnetic measurements form a Markov process with respect to the history of $\mathbf{j}(\mathbf{r}, t)$.

Algorithm. Particle filters are a class of sequential Monte Carlo methods which can be used for dynamically tracking the posterior probability density function $\pi(\mathbf{j}(t)|\mathbf{b}(t))$ for the neural currents; they exploit sequential importance sampling techniques for computing the two equations of Bayesian filtering, i.e. Bayes theorem and the Chapman-Kolmogorov equation:

$$\pi(\mathbf{j}(t)|\mathbf{b}(t)) = \frac{f(\mathbf{b}(t)|\mathbf{j}(t))\pi(\mathbf{j}(t))}{\pi(\mathbf{b}(t))} \tag{10}$$

$$\pi(\mathbf{j}(t+1)) = \int p(\mathbf{j}(t+1)|\mathbf{j}(t))\pi(\mathbf{j}(t)|\mathbf{b}(t))d\mathbf{j}(t) \tag{11}$$

where $f(\mathbf{b}(t)|\mathbf{j}(t))$ is the likelihood function and $p(\mathbf{j}(t+1)|\mathbf{j}(t))$ is the transition kernel of the neural current process, which are assumed to be known. At each time point t, a random sample is drawn, exploiting the transition kernel, from the prior density $\pi(\mathbf{j}(t+1))$ and each sampled point is assigned a weight through Bayes theorem, so that the weighted sample can be used to compute integral quantities according to the posterior density. Point estimates can thus be computed, like the conditional mean.

Parameters. Although particle filters are general enough to deal with non-Gaussian distributions, it is still reasonable to use Gaussian functions for the likelihood function and the transition kernel; the parameters the user has to choose are therefore

- the noise variance, or covariance matrix, within the likelihood function;
- the covariance matrix of the transition kernel.

However, we remark that the noise variance can be estimated from the data, while the covariance of the transition kernel can be determined on a physiological basis.

4. Comparison

We tested the three methods under several synthetic conditions; here we report three cases which highlight some of the limits and potentiality of the algorithms. For each condition, we show the original and the reconstructed time courses in Figures 1-5; here, BF stands for the EP beamformer, RAP for RAP-MUSIC and PF for particle filter. In Table 1 we give the localization errors provided by the three methods: while beamformers and RAP-MUSIC assume a stationary source configuration, and therefore the localization error is fixed across time, particle filters provide dynamical estimates of the source parameters: we computed both the average error (in the time window where the sources are active) and the localization error in correspondence of the maximum activity. Finally, let us remark that the three algorithms have been implemented using the same grid, and that the original source positions fall out of the grid points.

Table 1. The localization error (in mm) for the three algorithms under the different conditions; for particle filters we give the average error in the time window where sources are active, and, in parenthesis, the error in correspondence of the peak activity.

Localization error	EP beamformer	RAP-MUSIC	Particle filter
Condition I	3.5	3.5	6.0(1.3)
Condition II	2.0	2.0	3.0(1.0)
Condition III	2.8	2.8	7.9(2.8)

Condition I: orthogonal nearby sources. Two current dipoles with approximately the same position but different orientation are active in separate time windows. Under this very simple condition, particle filters and beamformers correctly recover the two sources, while RAP-MUSIC fails in estimating the orientations of the sources, and therefore their time courses as well.

Condition II: highly correlated sources. Two current dipoles located far from each other have approximately the same time courses. Particle filters and RAP-MUSIC correctly recover the original sources, while beamformers, if a 2-dimensional signal subspace is selected, fail in reconstructing the correct time courses.

Condition III: highly noisy data. MEG measurements produced by a single current dipole are corrupted by a strong amount of noise (Signal-to-Noise ratio $\simeq 2$ dB). The time course of the dipolar source is correctly recovered by all the three algorithms. The localization properties

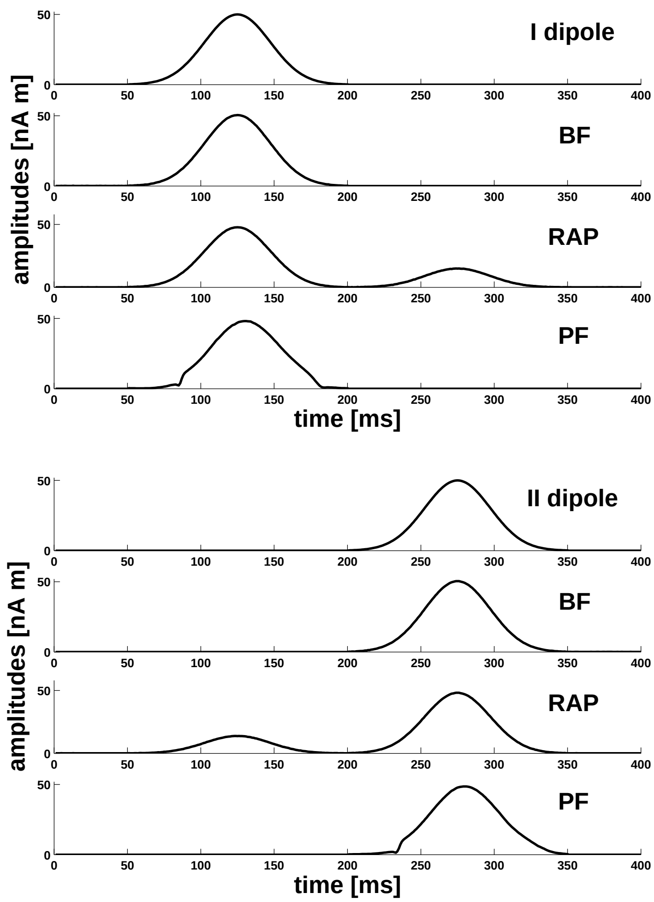

Figure 1. Condition I: amplitude time courses for the two source dipoles (top row of each panel) and the reconstructions provided by the three methods.

of beamformers and RAP-MUSIC do not seem to suffer much the high level of noise; particle filters, on the other hand, correctly estimate the source position around the peak of activity but the increased average error suggests that the variance of the estimate is higher.

5. Conclusions

The results of our analysis show that the three methods can be considered equivalent as far as the automaticity and the robustness with respect to noise are concerned. As expected, RAP-MUSIC and beamformers present some weaknesses in reconstructing sources with non-trivial spatial or temporal superpositions, while the general applicability conditions of particle filters allow reliable reconstructions even in such complicated situations. On the other hand, the results of particle filters are often slightly less accurate than the results obtained with the other two algorithms, provided that they work, which is at the moment the counterbalance of the more general assumptions.

References

[1] Hari R, Hämäläinen M, Ilmoniemi R J, Knuutila J, Lounasmaa O V 1993 Magnetoencephalography - theory, instrumentation and applications to noninvasive studies of the working human brain, *Reviews of Modern*

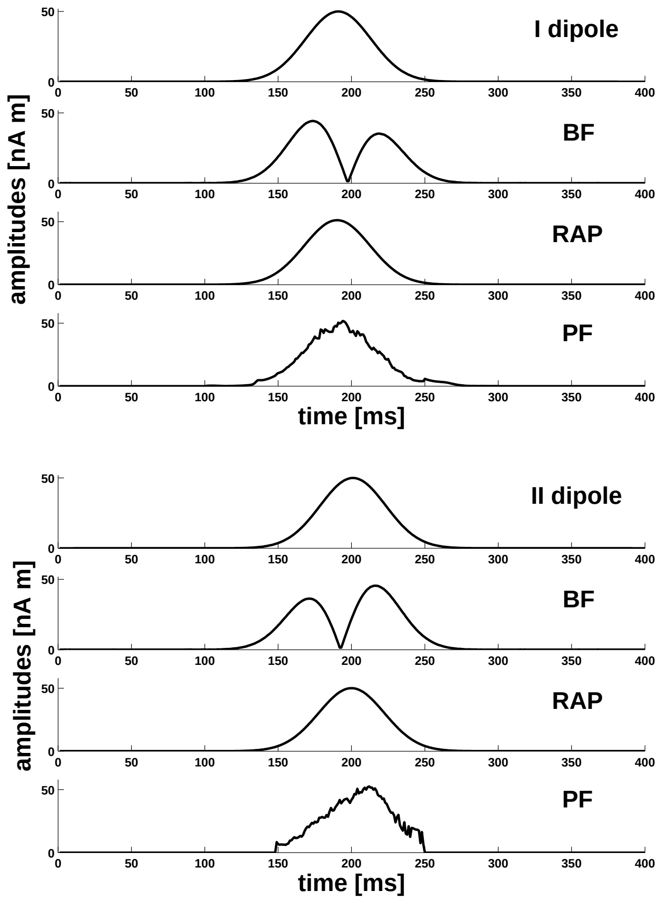

Figure 2. Condition II: amplitude time courses for the two source dipoles (top row of each panel) and the reconstructions provided by the three methods.

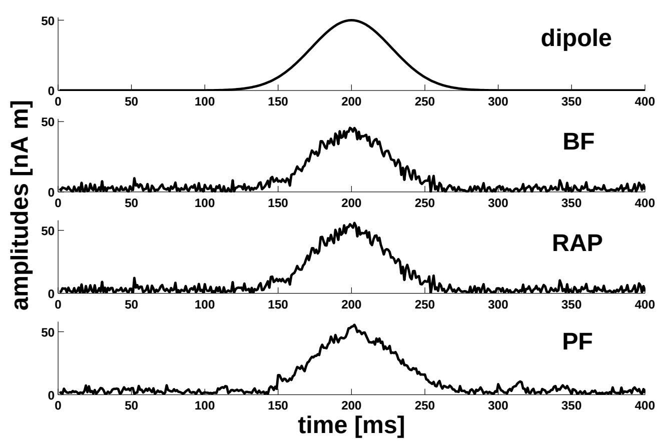

Figure 3. Condition III: amplitude time courses for the two source dipoles (top row) and the reconstructions provided by the three methods.

Physics, **65**, No.2

[2] Cantarella J, De Turck D, Gluck H 2001 The Biot - Savart operator for application to knot theory, fluid dynamics, and plasma physics, *Journal of Mathematical Physics*, **42**, 867-905

[3] Golub GH and Van Loan CF 1984 *Matrix Computation*, *The Johns Hopkins University Press*.

[4] Uutela K, Hämäläinen M, Salmelin R 1998 Global optimization in the localization of neuromagnetic sources, *IEEE Trans. Biomed. Eng.*, **45**, 716-723

[5] Marquardt DW 1963 An algorithm for least-squares estimation of nonlinear parameters, *J. Soc. Ind. Appl. Math.*, **11**, 431-441

[6] Hämäläinen M, Ilmoniemi R J 1999Interpreting magnetic fields of the brain: Minimun-norm estimates, *Med. Biol. Eng. Comp.*, **32**, 35-42

[7] Uutela K, Hämäläinen M, Somersalo E 1999 Visualization of magnetoencephalographic data using minimum current estimates, *NeuroImage*, **10**, 173-180

[8] Mosher J C, Leahy R M 1999 Source localization using recursively applied and projected (RAP) MUSIC, *IEEE Transactions on Signal Processing*, **47**, 332-340

[9] Van Veen B D, Buckley K 1988 Beamforming: A Versatile Approach to Spatial Filtering, *IEEE ASSP Magazine*, **5**, 4-24

[10] Sarvas J 1987 Basic mathematical and electromagnetic concepts of the biomagnetic inverse problem, *Phys. Med. Biol.*, **32**, 11-22.

[11] Somersalo E and Kaipio J P 2004 *Statistical and computational inverse problems*, *Springer Verlag*

[12] Sekihara K, Nagarajan S S, Poeppel D, Marantz A, Miyashita Y 2002 Application of an MEG Eigenspace Beamformer to Reconstructing Spatio-Temporal Activities of Neural Sources, *Human Brain Mapping*, **15**, 199-215

[13] Somersalo E, Voutilainen A , Kaipio J P 2003 Non-stationary magnetoencephalography by Bayesian filtering of dipole models, *Inverse Problems*, **19**, 1047-1063

[14] Sorrentino A, Parkkonen L, Piana M 2007 Particle filters: a new method for reconstructing multiple current dipoles from MEG data, *International congress series*, **1300** 173-176

[15] Pascarella A, Sorrentino A, Piana M, Parkkonen L 2007 Particle filters and RAP-MUSIC in MEG source modeling: a comparison, *International congress series*, **1300** 161-164