A-priori and a-posteriori shock capturing technique for high order CAT schemes for systems of conservation laws

R. Luobere, E. Macca,
C. Pares, G. Russo

University of Bordeaux, Malaga and Catania

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Lax-Wendroff

Let us consider scalar conservation laws

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the generalized L-W method is given by

$$u_i^{n+1} = u_i^n + \sum_{k=1}^{r} \frac{\Delta t^k}{k!} u_i^{(k)},$$

where the equailities $u_t = -au_x$ are used to approximate the time derivatives.

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where the equailities $u_t = -a u_x$ are used to approximate the time derivatives. This procedure can be extended to nonlinear systems by using the CK procedure Qui and Shu (2003) but the number of terms in the expression of the time derivatives increase exponentially.

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LAT-CAT

Zorio-Mulet-Baeza (2017) proposed the Approximate Taylor Method that using the Taylor approximations in time circumvented the Cauchy-Kovaleskya procedure

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The numerical methods obtained following the AT procedure of Zorio-Mulet-Baeza do not reduce to the LW methods for linear systems: they use $(4P+1)-$point stencils instead of $(2P+1)-$point ones. Carrillo-Parés (2019) proposed the Compact Approximate Taylor Method that properly extend the LW methods. It needs a stencil of $2P+1$ points and a family of interpolatory formulas based on the $2P+1$ points.

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$$u_i^{n+1} = u_i^n + \frac{\Delta t}{\Delta x}\left(F^P_{i-\frac{1}{2}} - F^P_{i+\frac{1}{2}}\right);$$

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where

$$F^P_{i\pm\frac{1}{2}} = f^{(0)}_{i\pm\frac{1}{2}} + \frac{\Delta t}{2} f^{(1)}_{i\pm\frac{1}{2}} + \ldots + \frac{\Delta t^{2P-1}}{(2P)!} f^{(2P-1)}_{i\pm\frac{1}{2}}$$

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$$f^{(k)}_{i+\frac{1}{2}} = \mathcal{I}\left(f^{(k)}_{i,-P+1}, \ldots, f^{(k)}_{i,P}\right) = \sum_{j=-P+1}^{P} \gamma^{0,\frac{1}{2}}_{P,j}\, f^{(k)}_{i,j},$$

$$f^{(k)}_{i-\frac{1}{2}} = \mathcal{I}\left(f^{(k)}_{i,-P}, \ldots, f^{(k)}_{i,P-1}\right) = \sum_{j=-P}^{P-1} \gamma^{0,\frac{1}{2}}_{P,j}\, f^{(k)}_{i,j}.$$

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$$f_{i,j}^{(k)} = \mathcal{D}_P^{k,j}\left(f_{i,j}^{k,*}, \Delta t\right) = \frac{1}{\Delta t^k}\sum_{r=-P+1}^{P} \gamma_{P,r}^{k,0} f_{i,j}^{k,n+r}$$

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with

$$u_{i,j}^{(k)} = -\mathcal{D}_P^{1,j}\left(f_{i,*}^{(k-1)}, \Delta x\right) = -\frac{1}{\Delta x}\sum_{s=-P+1}^{P} \gamma_{P,s}^{1,j} f_{i,s}^{(k-1)}$$

CAT scheme
CAT2, $P = 1$

$$\mathcal{S}_{i+\frac{1}{2}} = \{u_i^n, u_{i+1}^n\} \quad \text{and} \quad \mathcal{S}_{i-\frac{1}{2}} = \{u_{i-1}^n, u_i^n\}$$

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For all $j = 0, 1$

$$u_{i,j}^{(1)} = -\frac{f_{i+1}^n - f_i^n}{\Delta x} \quad \text{and} \quad u_{i-1,j}^{(1)} = -\frac{f_i^n - f_{i-1}^n}{\Delta x}$$

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$$f_{i,j}^{(1)} = \frac{f\left(u_{i+j}^n + \Delta t\, u_{i,j}^{(1)}\right) - f_{i+j}^n}{\Delta t}$$

$$f_{i-1,j}^{(1)} = \frac{f_{i+j}^n - f\left(u_{i-1+j}^n + \Delta t\, u_{i-1,j}^{(1)}\right)}{\Delta t}$$

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$$f^{(1)}_{i+\frac{1}{2}} = \frac{1}{2}\left(f^{(1)}_{i,0} + f^{(1)}_{i,1}\right) \quad \text{and} \quad f^{(1)}_{i-\frac{1}{2}} = \frac{1}{2}\left(f^{(1)}_{i-1,0} + f^{(1)}_{i-1,1}\right)$$

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Finally,

$$F^{1}_{i-\frac{1}{2}} = \frac{1}{2}\left(\tilde{f}^{(0)}_{i-1,0} + \tilde{f}^{(0)}_{i-1,1}\right) + \frac{\Delta t}{4}\left(\tilde{f}^{(1)}_{i-1,0} + \tilde{f}^{(1)}_{i-1,1}\right) =$$

$$= \frac{1}{4}\left(f^{n}_{i-1} + f^{n}_{i} + f\left(u^{n}_{i-1} + \Delta t\, u^{(1)}_{i-1,0}\right) + f\left(u^{n}_{i} + \Delta t\, u^{(1)}_{i-1,1}\right)\right),$$

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Finally,

$$\begin{aligned} F^1_{i-\frac{1}{2}} &= \frac{1}{2}\left(\tilde{f}^{(0)}_{i-1,0} + \tilde{f}^{(0)}_{i-1,1}\right) + \frac{\Delta t}{4}\left(\tilde{f}^{(1)}_{i-1,0} + \tilde{f}^{(1)}_{i-1,1}\right) = \\ &= \frac{1}{4}\left(f^n_{i-1} + f^n_i + f\left(u^n_{i-1} + \Delta t\, u^{(1)}_{i-1,0}\right) + f\left(u^n_i + \Delta t\, u^{(1)}_{i-1,1}\right)\right), \\ F^1_{i+\frac{1}{2}} &= \frac{1}{2}\left(\tilde{f}^{(0)}_{i,0} + \tilde{f}^{(0)}_{i,1}\right) + \frac{\Delta t}{4}\left(\tilde{f}^{(1)}_{i,0} + \tilde{f}^{(1)}_{i,1}\right) = \\ &= \frac{1}{4}\left(f^n_i + f^n_{i+1} + f\left(u^n_i + \Delta t\, u^{(1)}_{i,0}\right) + f\left(u^n_{i+1} + \Delta t\, u^{(1)}_{i,1}\right)\right), \end{aligned}$$

$$u^{n+1}_i = u^n_i + \frac{\Delta t}{\Delta x}\left(F^1_{i-\frac{1}{2}} - F^1_{i+\frac{1}{2}}\right);$$

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Smoothness indicators

$\varphi^1_{i+1/2}$ is an usual flux limiter function; Minmod, Superbee, Van Albada etc., *see Toro, Kemm ,Leveque.*

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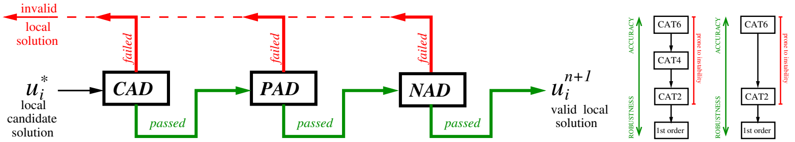
$$\omega_{i+1/2} = \frac{\omega^L_{i+\frac{1}{2}} \omega^R_{i+\frac{1}{2}}}{\omega^L_{i+\frac{1}{2}} + \omega^R_{i+\frac{1}{2}}},$$

$$\omega^L_{i+\frac{1}{2}} = \sum_{-p+1}^{-1} \left(f^n_{i+j+1} - f^n_{i+j} \right)^2 + \xi, \quad \omega^R_{i+\frac{1}{2}} = \sum_{1}^{p-1} \left(f^n_{i+j+1} - f^n_{i+j} \right)^2 + \xi.$$

MOOD

CATMOOD paradigm



(a) Criteria (b) Cascade

Figure: Left: Detection criteria of the MOOD technique for a candidate solution u_i^*. *Computer Admissible Detector (CAD)*, *Physical Admissible Detector (PAD)* and *Numerical Admissible Detector (PAD)* — Right: Order cascades of CAT schemes used in the MOOD procedure. Starting from the most accurate one, CAT6, downgrading to lower order schemes, and, at last to a 1st order accurate scheme employed to ensure robustness.

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2D Euler

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$$\mathrm{F}(U) = \begin{pmatrix} \rho u \\ \rho u^2 + p \\ \rho u v \\ (\rho e + p) u \end{pmatrix}, \quad \mathrm{G}(U) = \begin{pmatrix} \rho v \\ \rho u v \\ \rho u v + p \\ (\rho e + p) v \end{pmatrix}.$$

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$p = p(\rho, \varepsilon) = (\gamma - 1)(\rho e - \frac{1}{2}\|\mathrm{u}\|^2)$ which γ the adiabatic constant and $\mathrm{u} = (u, v)$.

Isentropic Vortex

Initial condition

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$u = u_\infty + \delta u,\ v = v_\infty + \delta v,\ T^* = T^*_\infty + \delta T^*$

$$\delta u = -y' \frac{\beta}{2\pi} \exp\left(\frac{1-r^2}{2}\right), \delta v \quad = x' \frac{\beta}{2\pi} \exp\left(\frac{1-r^2}{2}\right),$$

$$\delta T = -\frac{(\gamma-1)\beta^2}{8\gamma\pi^2} \exp\left(1-r^2\right),$$

Isentropic Vortex
Accuracy

2D Isentropic Vortex in motion - Rate of convergence

N	Rusanov-flux L^1 error	Rusanov-flux order	HLL L^1 error	HLL order	HLLC L^1 error	HLLC order	CATMOOD6 L^1 error	CATMOOD6 order
50 × 50	8.44×10^{-3}	—	8.44×10^{-3}	—	7.91×10^{-3}	-	8.48×10^{-3}	—
100 × 100	8.04×10^{-3}	0.07	8.04×10^{-3}	0.07	6.86×10^{-3}	0.21	3.77×10^{-3}	1.17
200 × 200	6.68×10^{-3}	0.27	6.67×10^{-3}	0.27	5.31×10^{-3}	0.37	2.40×10^{-7}	13.94
300 × 300	5.71×10^{-3}	0.36	5.71×10^{-3}	0.36	4.53×10^{-3}	0.39	2.06×10^{-8}	6.05
400 × 400	4.98×10^{-3}	0.47	4.98×10^{-3}	0.47	3.86×10^{-3}	0.55	3.52×10^{-9}	6.14
	Expected	1	Expected	1	Expected	1	Expected	6

N	CAT2 L^1 error	CAT2 order	CAT4 L^1 error	CAT4 order	CAT6 L^1 error	CAT6 order	ACAT6 L^1 error	ACAT6 order
50 × 50	7.94×10^{-3}	—	2.03×10^{-3}	—	8.46×10^{-4}	-	8.95×10^{-3}	—
100 × 100	2.55×10^{-3}	1.64	1.42×10^{-4}	3.83	1.56×10^{-5}	5.76	8.28×10^{-3}	0.11
200 × 200	6.12×10^{-4}	2.06	8.34×10^{-6}	4.09	2.41×10^{-7}	6.02	8.34×10^{-5}	9.95
300 × 300	2.69×10^{-4}	2.02	1.64×10^{-6}	4.02	2.09×10^{-8}	6.03	1.05×10^{-5}	5.14
400 × 400	1.52×10^{-4}	1.99	5.16×10^{-7}	4.01	3.68×10^{-9}	6.03	2.48×10^{-6}	4.93
	Expected	2	Expected	4	Expected	6	Expected	6

Isentropic Vortex
CPU time

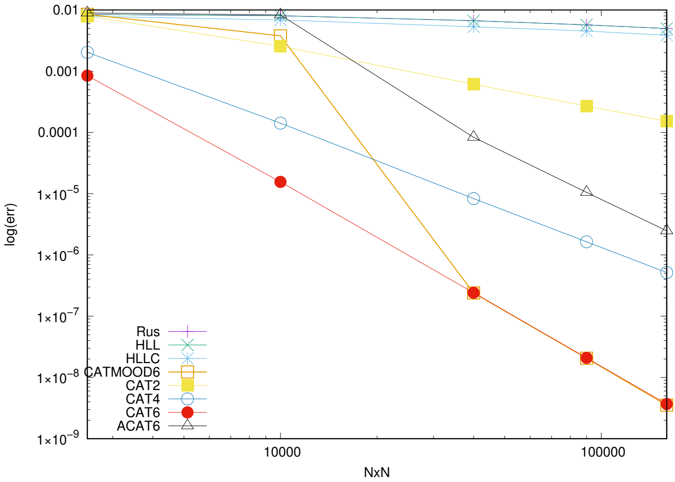


Figure: CPU time vs Errors

Sedov Blast Wave

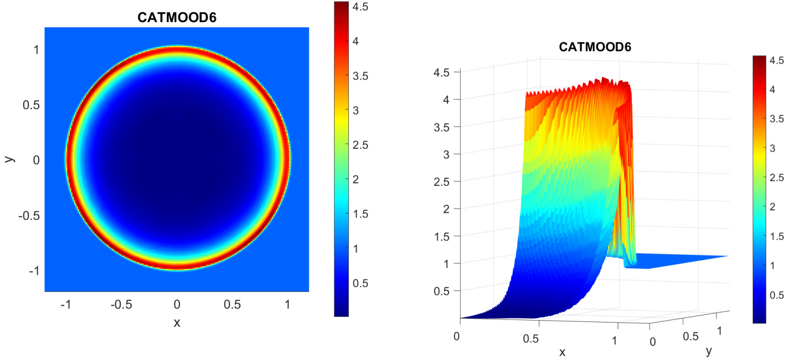
Density

The domain is given by $(x, y) \in [-1.2, 1.2]^2$ initially $(\rho^0, u^0, v^0, p^0, \gamma) = (1, 0, 0, 10^{-13}, 1.4)$. A total energy of $E_{total} = 0.244816$ is concentrated at the origin.

Sedov Blast Wave

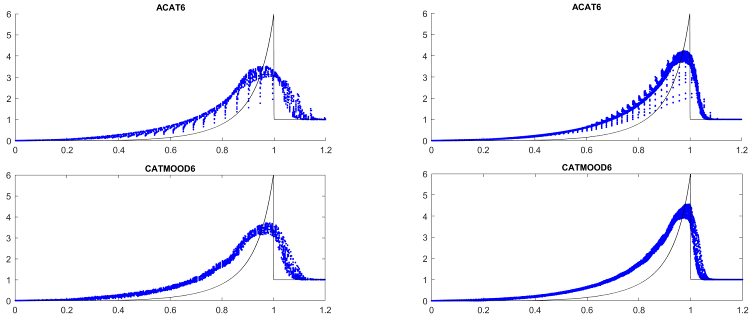
Density

The domain is given by $(x, y) \in [-1.2, 1.2]^2$ initially $(\rho^0, u^0, v^0, p^0, \gamma) = (1, 0, 0, 10^{-13}, 1.4)$. A total energy of $E_{total} = 0.244816$ is concentrated at the origin.



Sedov Blast Wave

Scatter plot



2D Riemann

Initial condition

Configuration 6

$\rho_2 = 2$	$u_2 = 0.75$	$\rho_1 = 1.5$	$u_1 = 0.75$
$v_2 = 0.5$	$p_2 = 1$	$v_1 = -0.5$	$p_1 = 1$
$\rho_3 = 1$	$u_2 = -0.75$	$\rho_4 = 3$	$u_4 = -0.75$
$v_3 = 0.5$	$p_2 = 1$	$v_4 = -0.5$	$p_4 = 1$

Configuration 11

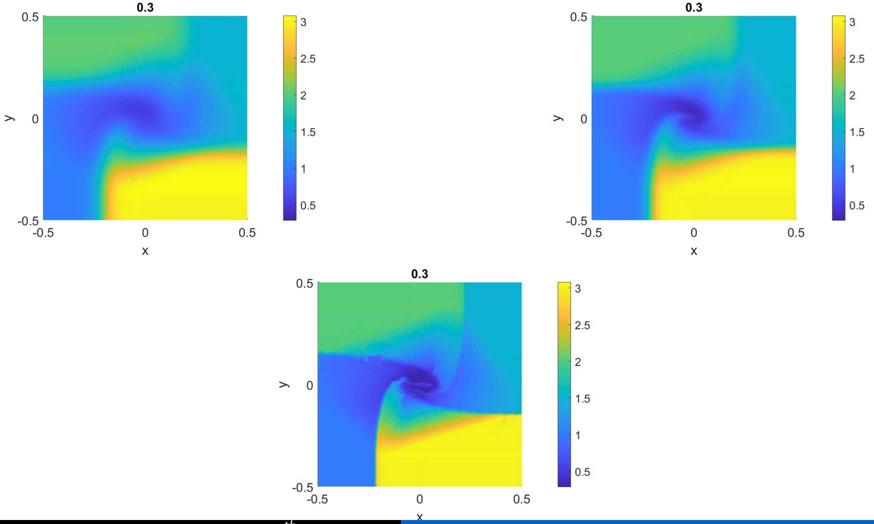
$\rho_2 = 0.5313$	$u_2 = 0.8276$	$\rho_1 = 1$	$u_1 = 0.1$
$v_2 = 0$	$p_2 = 0.4$	$v_1 = 0$	$p_1 = 1$
$\rho_3 = 0.8$	$u_2 = 0.1$	$\rho_4 = 0.5313$	$u_4 = 0.1$
$v_3 = 0$	$p_2 = 0.4$	$v_4 = 0$	$p_4 = 0.4$

Configuration 17

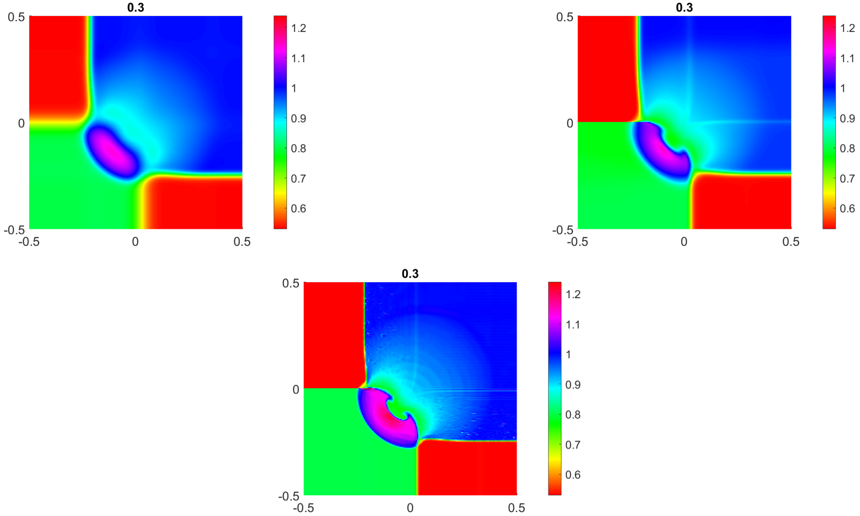
$\rho_2 = 2$	$u_2 = 0.$	$\rho_1 = 1$	$u_1 = 0$
$v_2 = -0.3$	$p_2 = 1$	$v_1 = -0.4$	$p_1 = 1$
$\rho_3 = 1.0625$	$u_2 = 0$	$\rho_4 = 0.5197$	$u_4 = 0$
$v_3 = 0.2145$	$p_2 = 0.4$	$v_4 = -1.1259$	$p_4 = 0.4$

2D Riemann

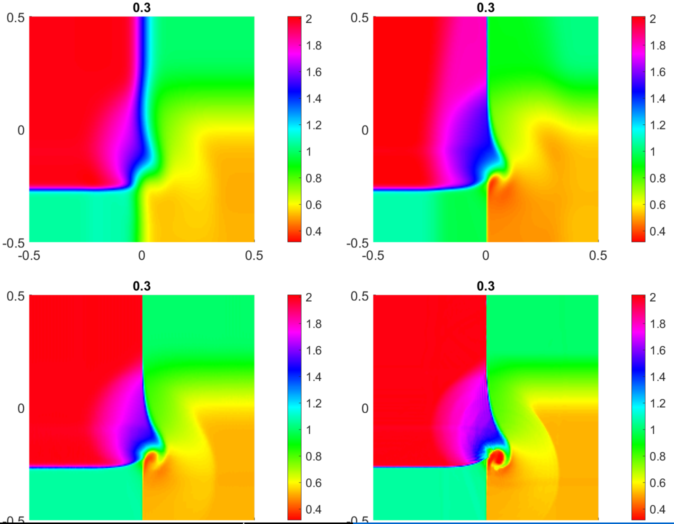
Configuration 6



2D Riemann
Configuration 11



2D Riemann
Configuration 17



Astrophysical jet

Mach 2000

$\Omega = [0,1] \times [-0.25, 0.25]$, $\gamma = 5/3$

$$(\rho^0, u^0, v^0, p^0) = \begin{cases} (5, 800, 0, 0.4127) & \text{if } x = 0 \text{ and } y \in [-0.05, 0.05], \\ (0.5, 0, 0, 0.4127) & \text{otherwise,} \end{cases}$$

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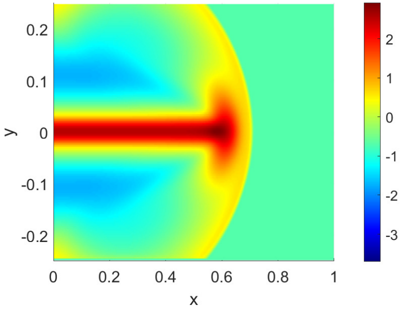

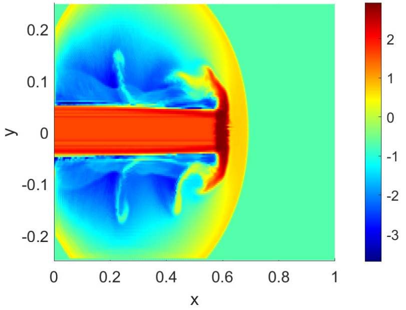

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Thanks for your attention